10. (a)

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $s$ | 0.71 | 1.00 | 0.71 | 0.00 | −0.71 |
| $v$ | 0.56 | 0.00 | −0.56 | −0.79 | −0.56 |
| $a$ | −0.44 | −0.62 | −0.44 | 0.00 | 0.44 |

(b) to the right at $t = 1$, stopped at $t = 2$, otherwise to the left

(c) speeding up at $t = 3$; slowing down at $t = 1, 5$; neither at $t = 2, 4$

11. (a) $v(t) = 3t^2 - 12t$, $a(t) = 6t - 12$

(b) $s(1) = -5$ ft, $v(1) = -9$ ft/s, speed $= 9$ ft/s, $a(1) = -6$ ft/s$^2$

(c) $v = 0$ at $t = 0, 4$

(d) for $t \geq 0$, $v(t)$ changes sign at $t = 4$, and $a(t)$ changes sign at $t = 2$; so the particle is speeding up for $0 < t < 2$ and $4 < t$ and is slowing down for $2 < t < 4$

(e) total distance $= |s(4) - s(0)| + |s(5) - s(4)| = |-32 - 0| + |-25 - (-32)| = 39$ ft

12. (a) $v(t) = 4t^3 - 4$, $a(t) = 12t^2$

(b) $s(1) = -1$ ft, $v(1) = 0$ ft/s, speed $= 0$ ft/s, $a(1) = 12$ ft/s$^2$

(c) $v = 0$ at $t = 1$

(d) speeding up for $t > 1$, slowing down for $0 < t < 1$

(e) total distance $= |s(1) - s(0)| + |s(5) - s(1)| = |-1 - 2| + |607 - (-1)| = 611$ ft

13. (a) $v(t) = -(3\pi/2)\sin(\pi t/2)$, $a(t) = -(3\pi^2/4)\cos(\pi t/2)$

(b) $s(1) = 0$ ft, $v(1) = -3\pi/2$ ft/s, speed $= 3\pi/2$ ft/s, $a(1) = 0$ ft/s$^2$

(c) $v = 0$ at $t = 0, 2, 4$

(d) $v$ changes sign at $t = 0, 2, 4$ and $a$ changes sign at $t = 1, 3, 5$, so the particle is speeding up for $0 < t < 1$, $2 < t < 3$ and $4 < t < 5$, and it is slowing down for $1 < t < 2$ and $3 < t < 4$

(e) 
$$\begin{aligned}\text{total distance} &= |s(2) - s(0)| + |s(4) - s(2)| + |s(5) - s(4)| \\ &= |-3 - 3| + |3 - (-3)| + |0 - 3| = 15 \text{ ft}\end{aligned}$$

14. (a) $v(t) = \dfrac{4 - t^2}{(t^2 + 4)^2}$, $a(t) = \dfrac{2t(t^2 - 12)}{(t^2 + 4)^3}$

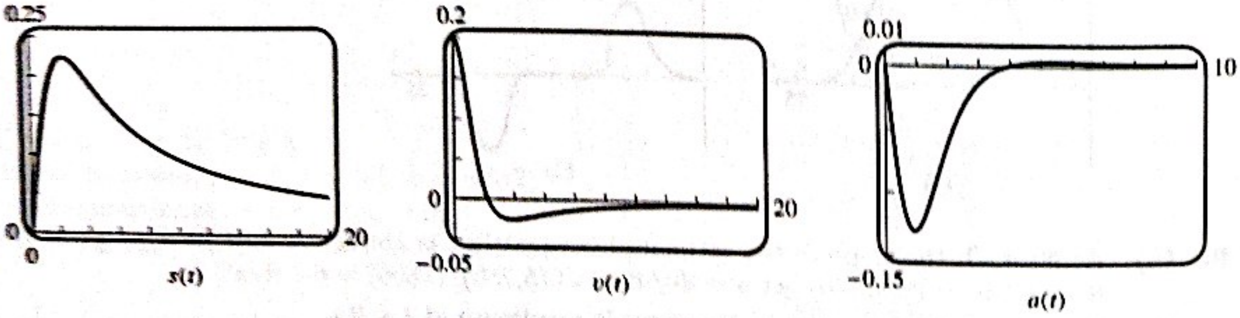
(b) $s(1) = 1/5$ ft, $v(1) = 3/25$ ft/s, speed $= 3/25$ ft/s, $a(1) = -22/125$ ft/s$^2$

(c) $v = 0$ at $t = 2$

(d) $a$ changes sign at $t = 2\sqrt{3}$, so the particle is speeding up for $2 < t < 2\sqrt{3}$ and it is slowing down for $0 < t < 2$ and for $2\sqrt{3} < t$

(e) total distance $= |s(2) - s(0)| + |s(5) - s(2)| = \left|\dfrac{1}{4} - 0\right| + \left|\dfrac{5}{29} - \dfrac{1}{4}\right| = \dfrac{19}{58}$ ft

15. $v(t) = \dfrac{5 - t^2}{(t^2 + 5)^2}$, $a(t) = \dfrac{2t(t^2 - 15)}{(t^2 + 5)^3}$

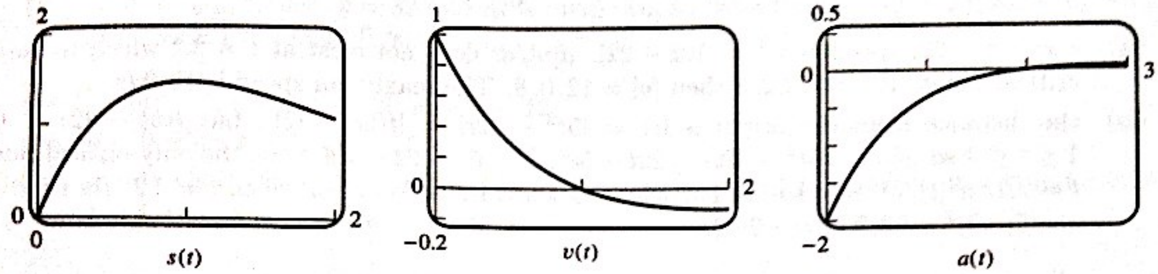



(a) $v = 0$ at $t = \sqrt{5}$

(b) $s = \sqrt{5}/10$ at $t = \sqrt{5}$

(c) $a$ changes sign at $t = \sqrt{15}$, so the particle is speeding up for $\sqrt{5} < t < \sqrt{15}$ and slowing down for $0 < t < \sqrt{5}$ and $\sqrt{15} < t$

**16.** $v(t) = (1-t)e^{-t}$, $a(t) = (t-2)e^{-t}$

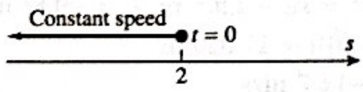



(a) $v = 0$ at $t = 1$

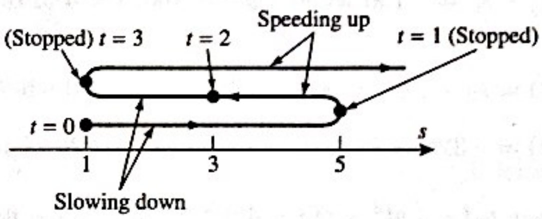
(b) $s = 1/e$ at $t = 1$

(c) $a$ changes sign at $t = 2$, so the particle is speeding up for $1 < t < 2$ and slowing down for $0 < t < 1$ and $2 < t$

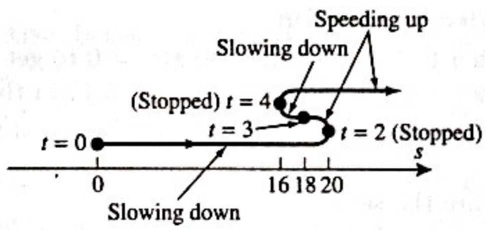
**17.** $s = -3t + 2$
$v = -3$
$a = 0$




**18.** $s = t^3 - 6t^2 + 9t + 1$
$v = 3(t-1)(t-3)$
$a = 6(t-2)$



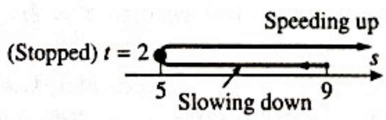

**19.** $s = t^3 - 9t^2 + 24t$
$v = 3(t-2)(t-4)$
$a = 6(t-3)$




**20.** $s = t + \dfrac{9}{t+1}$

$v = \dfrac{(t+4)(t-2)}{(t+1)^2}$

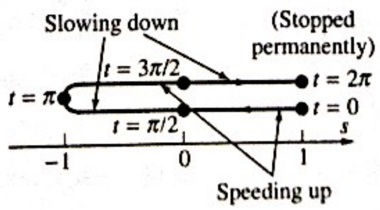
$a = \dfrac{18}{(t+1)^3}$




**21.** $s = \begin{cases} \cos t, & 0 \le t \le 2\pi \\ 1, & t > 2\pi \end{cases}$

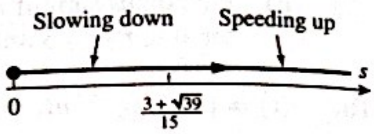
$v = \begin{cases} -\sin t, & 0 \le t \le 2\pi \\ 0, & t > 2\pi \end{cases}$

$a = \begin{cases} -\cos t, & 0 \le t < 2\pi \\ 0, & t > 2\pi \end{cases}$



**22.** $v(t) = \dfrac{5t^2 - 6t + 2}{\sqrt{t}}$ is always positive,

$a(t) = \dfrac{15t^2 - 6t - 2}{2t^{3/2}}$ has a positive root at $t = \dfrac{3 + \sqrt{39}}{15}$




**23.** **(a)** $v = 10t - 22$, speed $= |v| = |10t - 22|$. $d|v|/dt$ does not exist at $t = 2.2$ which is the only critical point. If $t = 1, 2.2, 3$ then $|v| = 12, 0, 8$. The maximum speed is 12 ft/s.

**(b)** the distance from the origin is $|s| = |5t^2 - 22t| = |t(5t - 22)|$, but $t(5t - 22) < 0$ for $1 \le t \le 3$ so $|s| = -(5t^2 - 22t) = 22t - 5t^2$, $d|s|/dt = 22 - 10t$, thus the only critical point is $t = 2.2$. $d^2|s|/dt^2 < 0$ so the particle is farthest from the origin when $t = 2.2$. Its position is $s = 5(2.2)^2 - 22(2.2) = -24.2$.

**24.** $v = -\dfrac{200t}{(t^2 + 12)^2}$, speed $= |v| = \dfrac{200t}{(t^2 + 12)^2}$ for $t \ge 0$. $\dfrac{d|v|}{dt} = \dfrac{600(4 - t^2)}{(t^2 + 12)^3} = 0$ when $t = 2$, which is the only critical point in $(0, +\infty)$. By the first derivative test there is a relative maximum, and hence an absolute maximum, at $t = 2$. The maximum speed is 25/16 ft/s to the left.

**25.** $s(t) = s_0 - \frac{1}{2}gt^2 = s_0 - 4.9t^2$ m, $v = -9.8t$ m/s, $a = -9.8$ m/s$^2$

**(a)** $|s(1.5) - s(0)| = 11.025$ m

**(b)** $v(1.5) = -14.7$ m/s

**(c)** $|v(t)| = 12$ when $t = 12/9.8 = 1.2245$ s

**(d)** $s(t) - s_0 = -100$ when $4.9t^2 = 100$, $t = 4.5175$ s

**26.** **(a)** $s(t) = s_0 - \frac{1}{2}gt^2 = 800 - 16t^2$ ft, $s(t) = 0$ when $t = \sqrt{\dfrac{800}{16}} = 5\sqrt{2}$

**(b)** $v(t) = -32t$ and $v(5\sqrt{2}) = -160\sqrt{2} \approx 226.27$ ft/s $= 154.28$ mi/h

**27.** $s(t) = s_0 + v_0t - \frac{1}{2}gt^2 = 60t - 4.9t^2$ m and $v(t) = v_0 - gt = 60 - 9.8t$ m/s

**(a)** $v(t) = 0$ when $t = 60/9.8 \approx 6.12$ s

**(b)** $s(60/9.8) \approx 183.67$ m

**(c)** another 6.12 s; solve for $t$ in $s(t) = 0$ to get this result, or use the symmetry of the parabola $s = 60t - 4.9t^2$ about the line $t = 6.12$ in the $t$-$s$ plane

**(d)** also 60 m/s, as seen from the symmetry of the parabola (or compute $v(6.12)$)

**28.** **(a)** they are the same

**(b)** $s(t) = v_0t - \frac{1}{2}gt^2$ and $v(t) = v_0 - gt$; $s(t) = 0$ when $t = 0, 2v_0/g$; $v(0) = v_0$ and $v(2v_0/g) = v_0 - g(2v_0/g) = -v_0$ so the speed is the same at launch ($t = 0$) and at return ($t = 2v_0/g$).

**29.** If $g = 32$ ft/s$^2$, $s_0 = 7$ and $v_0$ is unknown, then $s(t) = 7 + v_0t - 16t^2$ and $v(t) = v_0 - 32t$; $s = s_{max}$ when $v = 0$, or $t = v_0/32$; and $s_{max} = 208$ yields
$208 = s(v_0/32) = 7 + v_0(v_0/32) - 16(v_0/32)^2 = 7 + v_0^2/64$, so $v_0 = 8\sqrt{201} \approx 113.42$ ft/s.

**30.** **(a)** Use (6) and then (5) to get $v^2 = v_0^2 - 2v_0gt + g^2t^2 = v_0^2 - 2g(v_0t - \frac{1}{2}gt^2) = v_0^2 - 2g(s - s_0)$.

**(b)** Add $v_0$ to both sides of (6): $2v_0 - gt = v_0 + v$, $v_0 - \frac{1}{2}gt = \frac{1}{2}(v_0 + v)$; from (5) $s = s_0 + t(v_0 - \frac{1}{2}gt) = s_0 + \frac{1}{2}(v_0 + v)t$

**(c)** Add $v$ to both sides of (6): $2v + gt = v_0 + v$, $v + \frac{1}{2}gt = \frac{1}{2}(v_0 + v)$; from Part (b), $s = s_0 + \frac{1}{2}(v_0 + v)t = s_0 + vt + \frac{1}{2}gt^2$

**31.** $v_0 = 0$ and $g = 9.8$, so $v^2 = -19.6(s - s_0)$; since $v = 24$ when $s = 0$ it follows that $19.6s_0 = 24^2$ or $s_0 = 29.39$ m.